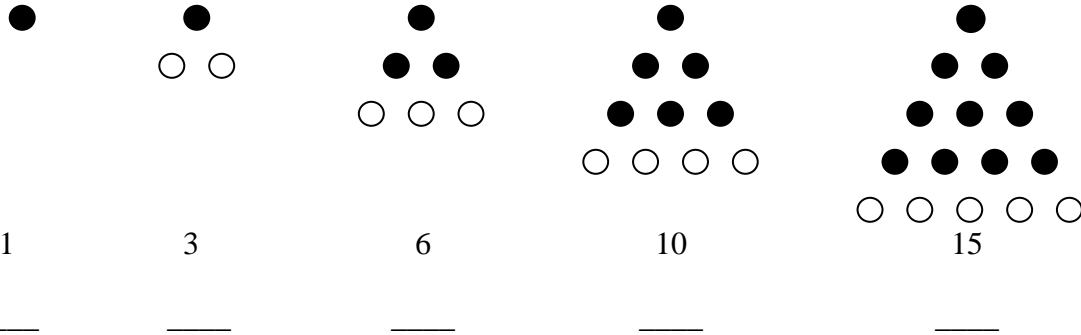


Date _____

Name _____

Figurate Numbers

- I. The **triangular numbers** are the whole numbers that are represented by certain triangular arrays of dots. The first five triangular numbers are shown. Look for a pattern that the numbers follow.



1. Complete the following table.

Number	Number of Dots (Triangular Number)
1	1
2	3
3	
4	
5	

2. Make a sketch to represent the sixth triangular number.

What is the sixth triangular number? _____

3. How many dots are there in the seventh triangular number? _____

Did you observe the following pattern? _____

- The 5th triangular number = The 4th triangular number + 5
- The 6th triangular number = The 5th triangular number + 6

4. Complete the following equations.

The 7th triangular number = _____ + _____

The _____ triangular number = The 7th triangular number + _____

The _____ triangular number = The _____ triangular number + 9

According to the pattern, the **nth triangular number = the (n - 1)th triangular number + _____**

Suppose T_1 represents the 1st triangular number, T_2 represents the 2nd triangular number, T_3 represents the 3rd triangular number, and so on to T_n , which represents the nth triangular number. The previous equations can be written as

$$T_2 = T_1 + 2$$

$$T_3 = T_2 + 3$$

$$T_4 = T_3 + 4$$

And so on.

The general statement would be: **$T_n = T_{n-1} + n$ (for $n \geq 2$)**

Note: If $n = 10$, then T_n represents the 10th triangular number and T_{n-1} represents the 9th triangular number.

5. Complete the following table.

$$T_1 = 1 = 1$$

$$T_1 = 1 = 1$$

$$T_1 = (1^2 + 1)/2 = 1$$

$$T_2 = 1 + 2 = 3$$

$$T_2 = 1 + 2 = 3$$

$$T_2 = (2^2 + 2)/2 = 3$$

$$T_3 = 3 + 3 = 6$$

$$T_3 = 1 + 2 + 3 = 6$$

$$T_3 = (3^2 + 3)/2 = 6$$

$$T_4 = 6 + 4 = 10$$

$$T_4 = 1 + 2 + 3 + 4 = 10$$

$$T_4 = (4^2 + 4)/2 = 10$$

$$T_5 = 10 + 5 = \underline{\hspace{2cm}}$$

$$T_5 = 1 + 2 + 3 + 4 + 5 = \underline{\hspace{2cm}}$$

$$T_5 = (5^2 + 5)/2 = \underline{\hspace{2cm}}$$

$$T_6 = \underline{\hspace{2cm}} + 6 = \underline{\hspace{2cm}}$$

$$T_6 = \underline{\hspace{2cm}} = \underline{\hspace{2cm}}$$

$$T_6 = (\underline{\hspace{1cm}}^2 + \underline{\hspace{1cm}})/2 = \underline{\hspace{2cm}}$$

$$T_7 = \underline{\hspace{2cm}} + \underline{\hspace{2cm}} = \underline{\hspace{2cm}}$$

$$T_7 = \underline{\hspace{2cm}} = \underline{\hspace{2cm}}$$

$$T_7 = (\underline{\hspace{1cm}}^2 + \underline{\hspace{1cm}})/2 = \underline{\hspace{2cm}}$$

$$T_8 = \underline{\hspace{2cm}} + \underline{\hspace{2cm}} = \underline{\hspace{2cm}}$$

$$T_9 = \underline{\hspace{2cm}} + \underline{\hspace{2cm}} = \underline{\hspace{2cm}}$$

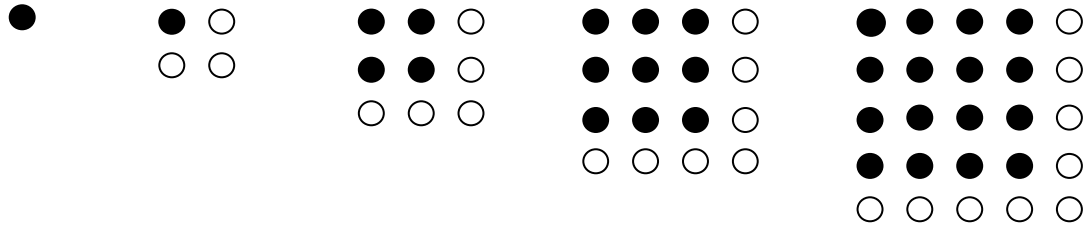
$$T_{10} = \underline{\hspace{2cm}} + \underline{\hspace{2cm}} = \underline{\hspace{2cm}}$$

6. Give two other general methods (formulas) for finding T_n :

$$T_n = \underline{\hspace{4cm}}$$

$$T_n = \underline{\hspace{4cm}}$$

II. The square numbers are the whole numbers that are represented by certain square arrays of dots. The first five square numbers are shown. Look for a pattern that the numbers follow.



S_1 _____

1. Complete the following table.

Number	Number of Dots (Square Number)
1	1
2	4
3	
4	
5	

2. Make a sketch to represent the sixth square number.

What is the sixth square number? _____

3. How many dots are there in the seventh square number? _____

4. Complete the following equations.

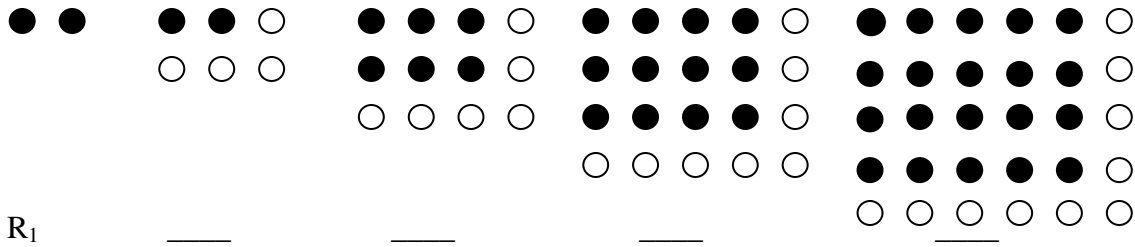
The 7th square number = (____)² or ____.

The ____ square number = 10² or ____.

Note: 10² = 10 x 10 = 100

According to the pattern, the **n**th square number = ____ squared or $S_n =$ _____.

III. The **rectangular numbers** are the whole numbers that are represented by certain rectangular arrays of dots. The first five rectangular numbers are shown. Look for a pattern that the numbers follow.



1. Complete the following table.

Number	Number of Dots (Rectangular Number)
1	2
2	6
3	
4	
5	

2. Make a sketch to represent the sixth rectangular number.

What is the sixth rectangular number? _____

3. How many dots are there in the seventh rectangular number? _____

Did you observe the following pattern? _____

- The 5th rectangular number = $5 \cdot 6$ or 30.
- The 6th rectangular number = $6 \cdot 7$ or 42.

4. Complete the following equations.

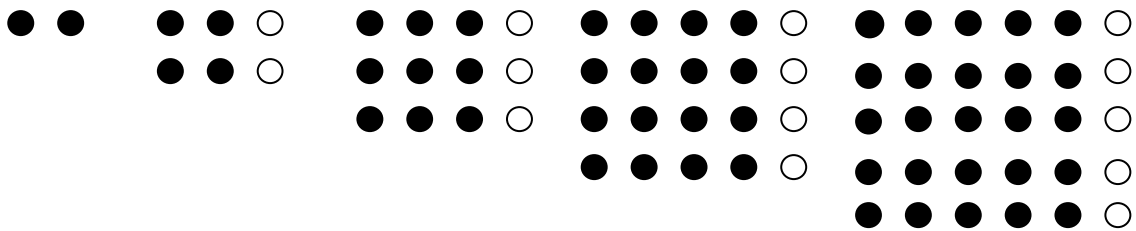
The 7th rectangular number = $7 \cdot$ _____ or _____.

The _____ rectangular number = $8 \cdot 9$ or _____.

The _____ rectangular number = _____ \cdot _____ or 110.

According to the pattern, the **nth rectangular number** = _____ \cdot _____.

5. There is another way to look at R_n numbers that you have not explored yet. Observe the following rectangular numbers:



R_1 _____ _____ _____ _____

6. Complete the following table.

Rectangular Number	Number of Dots (Rectangular Number)
R_1	$S_1 + 1$
R_2	$S_2 + 2$
R_3	
R_4	
R_5	

7. According to the pattern, $R_n =$ _____ $+$ _____.

Substitute the value you found for S_n into the formula you found above, then $R_n =$ _____ $+$ _____.

8. How is the last formula you obtained in #7 different from/the same as the last formula you obtained in #4 in this section?

9. How are triangular, square, and rectangular numbers related to one another?
